Parametric optics with whispering-gallery modes

Andrey B. Matsko, Vladimir S. Ilchenko, Rodolphe Le Targat, Anatoliy A. Savchenkov, and Lute Maleki

Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, California 91109-8099

ABSTRACT

We propose to fabricate a dielectric cavity sustaining high-Q whispering gallery modes from a periodically poled material possessing a quadratic nonlinearity to achieve an efficient interaction among the modes. We show that the periodical poling allows for compensation of both the material and the cavity dispersions that prohibits the nonlinear interaction otherwise. Such a cavity might be a basic element of a family of efficient nonlinear devices operating at a broad range of optical wavelengths.

Keywords: Whispering Gallery Modes, Parametric Frequency Conversion, Electro-Optical Modulation, Periodically Poled Lithium Niobate

1. INTRODUCTION

Whispering gallery modes (WGMs) in optical microresonators are attractive in nonlinear optics because of their small volumes and high quality factors.[1,2] An efficient parametric nonlinear interaction among the modes is possible if the cavity supporting WGMs is fabricated from a low loss $\chi^{(2)}$ nonlinear material. On the other hand, the nonlinear interactions are usually strongly forbidden by the momentum conservation law (phase matching condition) because modes of a dielectric cavity possessing rotational symmetry are orthogonal to each other in the momentum space. An interaction could only be possible if the symmetry of the system is broken or modified.

A way of such a modification was recently suggested for a strongly-nondegenerate three-wave mixing.[3,4] A resonant interaction of light confined in three optical WGMs, and a microwave field, was achieved by engineering the geometry of a microwave resonator coupled to a toroidal $LiNbO_3$ optical cavity. A new kind of electro-optic modulator and photonic receiver based on this interaction was realized.[3–9]

Here we propose an approach to the realization of a parametric interaction among different WGMs using a disc cavity produced from a periodically poled nonlinear material, e.g. periodically poled lithium niobate (PPLN).[10] We show that it is possible to achieve coupling among light waves as well as among light and microwaves adjusting step of the poling in an appropriate way. High quality factors of WGMs result in low threshold for parametric oscillations in the system.

As an example, we calculate the structure of the cavity poling necessary to create a triply-resonant optical parametric oscillator that converts a pump photon ($\lambda_0 = 1064$ nm) into signal and idler photons ($\lambda_s = \lambda_i = 2128$ nm). Such a parametric oscillator might have a sub-microWatt threshold power for realistic parameters. We show as another example a possibility of a parametric interaction among running light and microwave radiation in a periodically poled WGM cavity. Such a modulator has easier tunability compared with all-resonant WGM electro-optical modulators with standing microwave.

Our results are promising for various applications in nonlinear and quantum optics ranging from optical frequency conversion, optical modulation, and photonic reception, to production of nonclassical states of light, quantum nondemolition measurements, and quantum computing.

Send correspondence to A.B.Matsko: Andrey.Matsko@jpl.nasa.gov

2. PARAMETRIC CONVERSION IN A PPLN WGM CAVITY

An efficient wave mixing with WGMs might be hindered by two intrinsic properties of a dielectric WGM cavity: i) frequency dependent dispersion of the host material of the dielectric cavity, and ii) the dispersion introduced by the internal geometrical mode structure. The frequency of a high order TE WGM may be estimated from

$$\frac{2\pi R}{\lambda}\sqrt{\epsilon(\lambda)}+\sqrt{\frac{\epsilon(\lambda)}{\epsilon(\lambda)-1}}=\nu+\alpha_q\left(\frac{\nu}{2}\right)^{1/3}+\frac{3\alpha_q^2}{20}\left(\frac{2}{\nu}\right)^{1/3}, \tag{1}$$

where λ is the wavelength in vacuum, ν is the mode order, $\epsilon(\lambda)$ is the susceptibility of the nonlinear material, R is the radius of the cavity, and α_q is the qth root of the Airy function, $Ai(-z)$, which is equal to 2.338, 4.088, and 5.521 for $q = 1, 2, 3$, respectively.

We assume that the cavity is fabricated from a commercial flat Z-cut $LiNbO_3$ substrate, so TE modes correspond to the extraordinary waves in the material. For example, let us consider a degenerate optical parametric oscillator (OPO) pumped at $\lambda_p = 1064$ nm. Susceptibilities for the pump, signal, and idler waves are $\epsilon(\omega_p) = 4.657$, $\epsilon(\omega_s) = \epsilon(\omega_i) = 4.513$ respectively.

To achieve phase-matching for such an OPO in a bulk, congruent $LiNbO_3$, poling periods Λ have to obey the following condition[10]

$$k_p - k_s - k_i - \frac{2\pi}{\Lambda} = 0, \tag{2}$$

where k_p, k_s, and k_i are the wave vectors of the pump, the signal and the idler, respectively. For the degenerate case discussed in the paper the poling period is determined by

$$\Lambda = \lambda_p \frac{m}{\sqrt{\epsilon_p}-\sqrt{\epsilon_s}} \approx 31.67\ \mu m\ . \tag{3}$$

where m is an integer number. In the following we assume that $m = 1$.

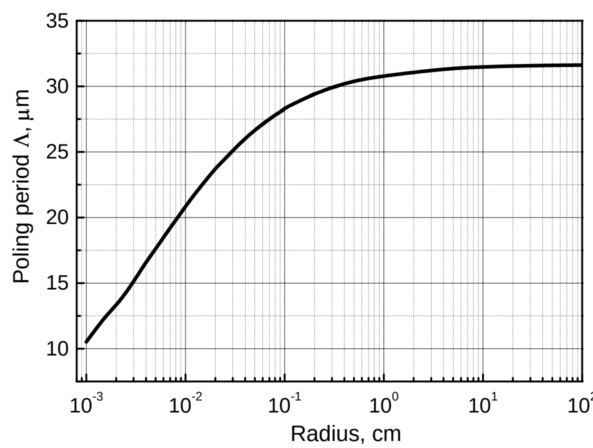

Figure 1. Dependence of poling period Λ on the WGM cavity radius. It is easy to see that for a large cavity radius the period coincides with the period in a bulk medium, while for smaller radius the period decreases.

For a WGM cavity fabricated from the same material the poling period is different because of the dependence of the mode dispersion on the cavity geometrical parameters. The smaller is the cavity the shorter is the period (see Fig. 1). Moreover, there is no guarantee that the pump and the signal frequencies are both resonant with the cavity modes. To characterize this possibility we assume that the pump is resonant with a cavity mode and introduce effective detuning

$$\frac{\Delta}{\gamma_s} = \left|\frac{\omega_p}{\widetilde{\omega}_s} - 2\right| Q_s, \tag{4}$$

where γ_s and Q_s are the linewidth and the quality factor of the signal mode $Q_s = \omega_s/(2\gamma_s)$, $\widetilde{\omega}_s$ is the frequency of the cavity mode that minimizes Δ. If $\Delta/\gamma_s < 1$ we may say that the signal frequency is resonant with the cavity mode. The dependence of the detuning on the cavity radius in not trivial. The averaged dependence is presented in Fig. 2 for $Q_s = 10^7$. We conclude from this calculation that the smaller is the cavity the less is the probability that the cavity sustain both resonant frequencies ω_p and $\omega_p/2$. The cavity radius should exceed 2 mm to have this condition fulfilled, as it follows from Fig. 2.

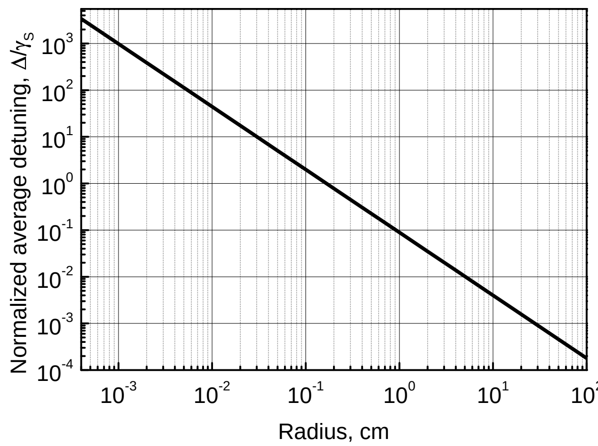

Figure 2. Averaged detuning of the signal frequency from the nearest WGM of a dielectric cavity vs cavity radius. The pump field ($\lambda_p = 1.064$ μm) is resonant with a mode of the $LiNbO_3$ cavity.

However, condition $\Delta/\gamma_s < 1$ is not critical even for smaller cavities. The frequency difference may be compensated by applying a DC bias field to the cavity. The DC field moves differently modes of different frequencies, which results in a possibility of frequency matching in the system.

Let us consider a cavity with radius $R = 0.64$ mm and study the main mode sequence ($\alpha_q = 2.338$). It worth noting that fabrication of high-Q cavities with such a value of the cavity radius has been recently realized for mm-wave electro-optical modulator.[9] The mode resonant with the pump field has index $\nu_p = 8156$. For the signal and idler modes $\nu_s = \nu_i = 4004$. Because $\nu_p - \nu_s - \nu_i = 148 > 1$ there is no parametric interaction in the system unless the nonlinear index of refraction of the cavity material is modulated with the period determined by the number $\nu_\chi = \nu_p - \nu_s - \nu_i$. This periodicity also results in phase-matching for degenerate parametric frequency downconversion with 1.056 μm, 1.049 μm, 1.041μm, etc, pump radiation.

The approximate modulation period is 27.3 μm. It is easy to achieve such a modulation by a periodic poling of the cavity material.[10] The volumes of the pump mode, the signal/idler mode, and modes' overlap are equal to 1.7×10^{-7} cm^3, 4×10^{-7} cm^3, and 1.5×10^{-7} cm^3 respectively (see the mode profiles Fig. 3). Note that the mode volumes can be also estimated using the asymptotic expression $V_p \approx 2\pi R \times 2R(2\pi/\nu_p)^{1/2} \times (R/\nu_p^{2/3})$.

The radius of the cavity is contained only in the ratio R/λ in Eq.(1). Therefore, the phase matching established by the periodical poling of the cavity material is stable with respect to the radius change, and may be compensated by fine tuning of the pump laser frequency.

Let us now estimate the threshold for parametric oscillations. The interaction energy between the pump and signal modes may be written as

$$\mathcal{V} = \int\limits_V \chi^{(2)} E_p E_s^2 \, dV, \tag{5}$$

where $\chi^{(2)}$ is varying in space nonlinearity of the cavity, E_p and E_s are the pump and signal mode amplitudes

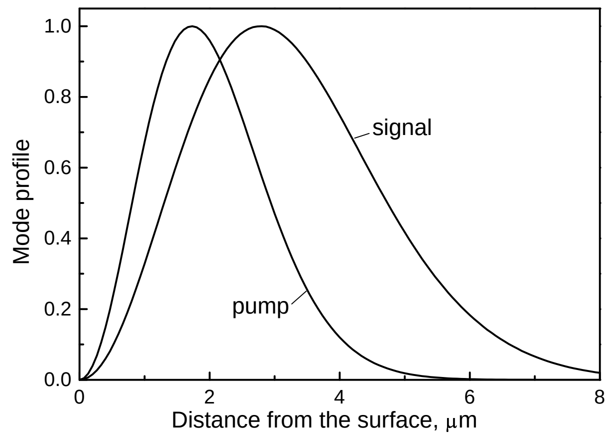

Figure 3. Profiles of the field distribution inside a dielectric cavity. Zero coordinate corresponds to the cavity boundary. In this model we neglect the evanescent field outside the cavity, which is a reasonable approximation for high-Q WG modes.

(the signal mode coincides with the idler mode), V is the volume of the cavity. We present mode amplitudes as

$$E_p = \sqrt{\frac{2\pi\hbar\omega_p}{\epsilon_p V_p}}\Psi_p(r)e^{i\nu_p\phi}a_p e^{-i\omega_p t} + adjoint, \tag{6}$$

$$E_s = \sqrt{\frac{2\pi\hbar\omega_s}{\epsilon_s V_s}}\Psi_s(r)e^{i\nu_s\phi}a_s e^{-i\omega_s t} + adjoint, \tag{7}$$

where a_i and $a_i^\dagger$ $(i = p, s)$ are annihilation and creation operators for the mode, $\Psi_p(r)$ and $\Psi_s(r)$ are the mode spatial profiles normalized such that $V_i = \int_V |\Psi_i(r)|^2 dV$ $(i = p, s)$, ω_p and ω_s are the mode frequencies, ϵ_p and ϵ_s are the susceptibilities of the material. The problem of quantization of electromagnetic waves in dielectrics was discussed in.[11] We also assume that the nonlinearity of the medium is modulated such that it has a Fourier component that matches the signal and pump modes: $\chi^{(2)} \to 2\tilde{\chi}^{(2)}\cos\left[(\nu_p - 2\nu_s)\phi\right]$.

We now write the interaction Hamiltonian in slowly varying amplitude and phase approximation

$$H = \hbar g((a_s^\dagger)^2 a_p + a_p^\dagger a_s^2), \tag{8}$$

where the coupling constant is

$$g = 2\pi\omega_s \frac{\tilde{\chi}^{(2)}}{\epsilon_s}\frac{V_{pss}}{V_s}\sqrt{\frac{2\pi\hbar\omega_p}{\epsilon_p V_p}}, \tag{9}$$

$V_{pss} = \int_V \Psi_p \Psi_s^2 dV < V_p, V_s$ is the mode overlap integral.

Using this Hamiltonian we derive equations of motion

$$\dot{a_p} = -\gamma_p a_p - iga_s^2 + F_p, \tag{10}$$

$$\dot{a_s} = -\gamma_s a_s - 2iga_s^\dagger a_p + F_s, \tag{11}$$

where F_p, F_s are the Langevin forces, γ_p and γ_s are pump and signal decay rates respectively. The expectation value $\langle F_p\rangle$ describes pumping from outside of the system. We can write the expression $|F_p|^2/\gamma_p^2 = 4W_pQ_p/(\hbar\omega_p^2)$, where $Q_p = \omega_p/(2\gamma_p)$ is the mode quality factor, W_p is the power of the pump radiation (in vacuum).

We solve Eqs. (10) and (11) in steady state neglecting quantum fluctuations.

$$a_p = -\frac{ig}{\gamma_p}a_s^2 + \frac{F_p}{\gamma_p}, \tag{12}$$

$$a_s = -\frac{2ig}{\gamma_s}a_s^* a_p. \tag{13}$$

Substituting (12) into (13) we obtain

$$a_s\left(1+\frac{2g^2}{\gamma_s\gamma_p}|a_s|^2\right) = -\frac{2ig}{\gamma_s}a_s^*\frac{F_p}{\gamma_p}. \tag{14}$$

Eq.(14) contains information about amplitude and phase of the generated signal. Introducing $F_p = |F_p|\exp(i\phi_{Fp})$ and $a_s = |a_s|\exp(i\phi_s)$, we find the phase and the photon number for the second harmonic generated in the system

$$2\phi_s = \phi_{Fp} - \frac{\pi}{2}, \tag{15}$$

$$|a_s|^2 = \frac{\gamma_p\gamma_s}{2g^2}\left(\frac{2g}{\gamma_s\gamma_p}|\langle F_p\rangle| - 1\right), \tag{16}$$

that results in threshold condition for the parametric oscillation

$$W_p \geq \frac{\epsilon_p\epsilon_s^2}{512\pi^3(\tilde{\chi}^{(2)})^2}\left(\frac{V_s}{V_{pss}}\right)^2\frac{\omega_p V_p}{Q_s^2 Q_p}. \tag{17}$$

For realistic parameters ($V_{pss}/V_s = 0.5$, $V_p = 2\times10^{-7}$ cm^3, $\epsilon_p = 4.6$, $\epsilon_s = 4.5$, $\tilde{\chi}^{(2)} = 10^{-7}$ CGSE, $\omega_p = 2\times10^{15}$ s^{-1}, $Q_s \simeq Q_p \simeq 10^7$) threshold value is 0.1 μW, which is orders of magnitude less than the state of the art OPO threshold power, e.g. 0.5 mW for similar wavelengths.[12]

We expect that the toroidal cavity is advantageous over the total-internal-reflection cavity used in optical parametric oscillators pumped at 1064 nm.[13] The oscillation threshold depends on the mode volume and the mode overlap integral. We can reduce the mode volume and increase the integral in WG mode cavity.

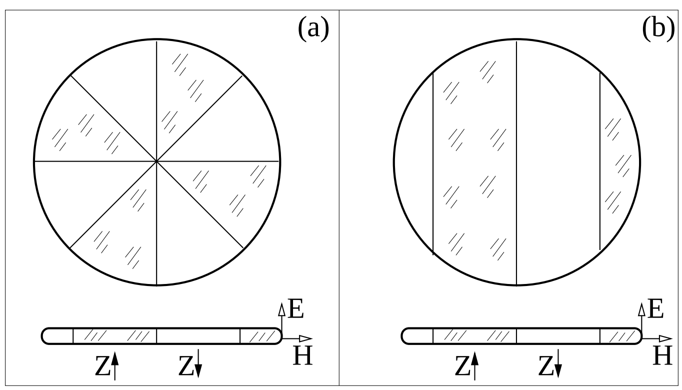

Figure 4. Examples of material poling to achieve a phase-matching for WG modes: (a) Poling symmetric with respect to the cavity center; (b) stripe-like poling.

Finally, let us discuss the possibilities for fabrication of the nonlinear cavities. The optimum poling geometry is symmetric with respect to the center of the cavity (Fig. 4a). When the nonlinear index coefficient is modulated with a periodic sign reversal the Fourier coefficient for the first harmonic is about $2/\pi$, as in a periodically poled bulk material. The dependence of the Fourier coefficients on the inverse poling period is shown in Fig. 5, where we average the discrete spectrum and show its envelop function.

Fabrication of the centrally symmetric poling is a difficult task. It seems to be much easier to use a slice of the commercially available periodically poled $LiNbO_3$. The cavity will have poled stripes, instead of sectors (see Fig. 4b). For such a poling a wave that travels close to the cavity surface sees nonlinear grating with changing period. An envelop function for the Fourier decomposition for such a grating is shown in Fig. 6. Because the grating does not have a fixed step with respect to the electromagnetic wave, the maximum Fourier component is less than in the ideal case (symmetric poling). This leads to an increase of the oscillation threshold compared

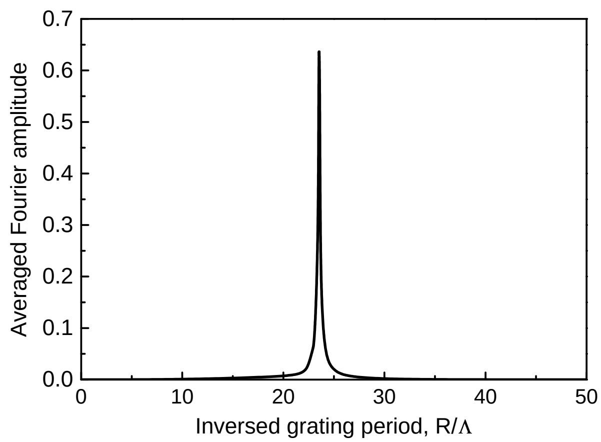

Figure 5. The envelop function for the amplitude of fourier coefficients for the poling shown in Fig. 4a.

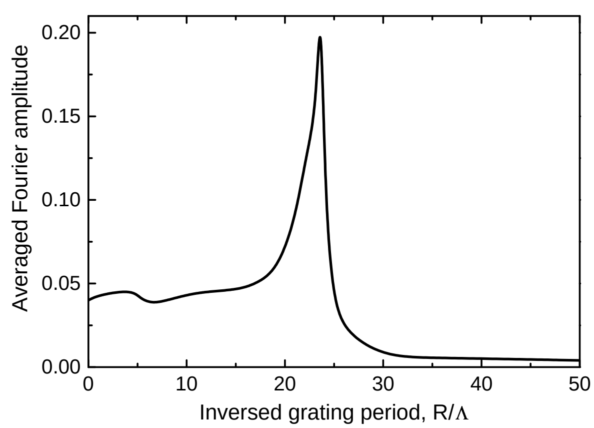

Figure 6. The envelop function for the amplitude of fourier coefficients for the poling shown in Fig. 4b.

with periodic poling (11 times in the case considered). However, the spectrum of the grating is broad enough to simplify work with the cavity and make possible multi-frequency parametric oscillations in the system. Non-equidistant poling stripes may, in principle, create a more periodic poling and change the spectrum. However, cutting of a disc from a sample with complicated poling structure is not an easy task.

It is worth mentioning that the nonlinear system discussed above may also be used to produce nondegenerate parametric interactions. Because of the low threshold the system may also be "reversed" and used as a detector of long wavelength radiation via its direct upconversion into light. The maximum wavelength of the radiation is determined by the cavity size. To maintain low radiative losses the order of the cavity mode should be sufficiently large. For example, to detect 10 μm radiation the cavity should have a radius of, at least, 3.5 mm.

3. TRAVELLING WAVE ELECTRO-OPTICAL MODULATOR WITH A PPLN WGM CAVITY

A lot of communication applications require devices capable of receiving, transforming and processing signals in millimeter wavelength domain. Electro-optic modulators (EOMs) based on electromagnetic wave interaction in nonlinear WGM optical cavities are promising here.[3–9]

Optical cavity-based WGM EOMs allow for reduction of microwave operating power compared with the commercially available devices. The core of the WGM modulator is a WGM cavity made with a second-order nonlinearity material, such as $LiNbO_3$ or $LiTaO_3$. Quality factors of WGMs may be large enough because of small optical losses in the materials. Even a small voltage applied across the area of confinement of the optical field is enough to induce a change in the frequency of the WGM with a magnitude comparable to its linewidth. This forms the basis for an efficient modulation.

The principle of EOM operation is based on three wave mixing, similarly to the parametric process described in the previous Section. The only difference is that the idler mode has a wavelength in a millimeter range now and, in general, the idler needs metal waveguide system to be confined in the interaction region. To achieve the modulation one have to optically pump a WGM mode and simultaneously send a microwave radiation to the system. The efficient optical-microwave interaction may occur when the frequency difference or sum of the optical and microwave pumping coincides with resonant frequency of another, signal, WGM mode. Such an interaction, however, may be forbidden by the phase matching conditions. A solution of the problem based on microwave cavity engineering was proposed in.[3–9]

Here we show that periodical poling of $\chi^{(2)}$ materials may be used for fabrication of WGM EOMs, instead of the specially shaped microwave cavities that were previously utilized for efficient modulation of light. The travelling wave EOM has easier tunability than the standing wave modulator. Disadvantage of the travelling wave WGM EOM compared with WGM EOM utilizing microwave resonator is absence of accumulation of the microwave power in the interaction region and, hence, weaker modulation for the same values of the microwave pump power sent to the system.

Usefulness of periodically poled materials (quasi-phase matching) for light modulation was recognized more than a decade ado.[14] The technique was further developed recently.[15–18] Domain reversal period Λ necessary to achieve a phase matching between light and microwaves in a planar geometry is similar to Eq.(2):

$$\frac{2\pi}{\Lambda} = \frac{\omega_M}{c}(\sqrt{\epsilon_M} - \sqrt{\epsilon_p}) + \frac{\omega_s}{c}(\sqrt{\epsilon_s} - \sqrt{\epsilon_p}), \tag{18}$$

where we assumed that $\omega_p = \omega_s + \omega_M$, ω_p and ω_s are frequencies of optical pumping and signal, ω_M is the microwave frequency; ϵ_p, ϵ_s, and ϵ_M are the pump, signal, and microwave susceptibilities of the medium. Eq. (18) may also be written as (see,[15] for example)

$$\Lambda = \frac{2\pi}{\omega_M} \frac{1}{\dfrac{1}{v_m} - \dfrac{1}{v_g}}, \tag{19}$$

where $v_m = c/\sqrt{\epsilon_M}$ is the phase velocity of microwaves, v_g is group velocity for the light. To derive this expression we assumed that $\omega_p \simeq \omega_s \gg \omega_M$.

Group velocity for the light travelling in WGM cavity may be estimated from Eq. (1). To do it we use an analogy between WGM and a mode of a ring fiber cavity. For such a cavity

$$\frac{2\pi R}{\lambda}\sqrt{\epsilon_f(\lambda)} = \nu, \tag{20}$$

where $\epsilon_f(\lambda)$ is an effective susceptibility of the fiber that takes into account both material and geometrical dispersions of the fiber. The group velocity may be introduced as

$$\frac{\omega_{\nu+1}}{c}\sqrt{\epsilon_f(\omega_{\nu+1})} = \frac{\omega_\nu}{c}\sqrt{\epsilon_f(\omega_\nu)} + \frac{1}{v_g}(\omega_{\nu+1} - \omega_\nu), \tag{21}$$

where $\omega_\nu = 2\pi c/\lambda_\nu$. Comparison of Eqs. (20) and (21) gives us

$$v_g = R(\omega_{\nu+1} - \omega_\nu). \tag{22}$$

Neglecting material dispersion we derive an expression for an effective group velocity for the main WGM sequence

$$v_g \approx \frac{c}{\sqrt{\epsilon_p}}\left[1 + \frac{0.62}{\nu_p^{2/3}}\right]. \tag{23}$$

Group velocity is a little bit larger than the phase velocity because the wave with larger frequency has a longer round trip path. We may neglect by the dependence of the group velocity on the mode order for large ν_p we are interested in, and rewrite (19) as

$$\Lambda = \frac{2\pi}{\omega_M} \frac{c}{\sqrt{\epsilon_M} - \sqrt{\epsilon_p}}. \tag{24}$$

Assuming that $\sqrt{\epsilon_M} \approx 4.2$, $\sqrt{\epsilon_p} \approx 2.1$ ($\lambda_p = 1.55$ μm), and $\omega_M = 2\pi \times 100$ GHz, we find $\Lambda = 0.15$ cm. It worth noting that the value of $\sqrt{\epsilon_M}$ vary depending on the microwave waveguide properties.

For comparably low modulation frequency no periodical poling is necessary. Without periodical poling the optimum interaction length for the light and microwaves is $\Lambda/2$, where Λ is determined by Eq. (24). For example, for experimental setup discussed in,[7] with $R = 2.3$ mm and $\omega_M = 2\pi \times 10$ GHz, the interaction region should be 0.75 cm long, which is approximately a half of the WGM cavity rim length. The periodical poling becomes important if we would like to achieve a modulation with a higher frequency in the same large cavity.

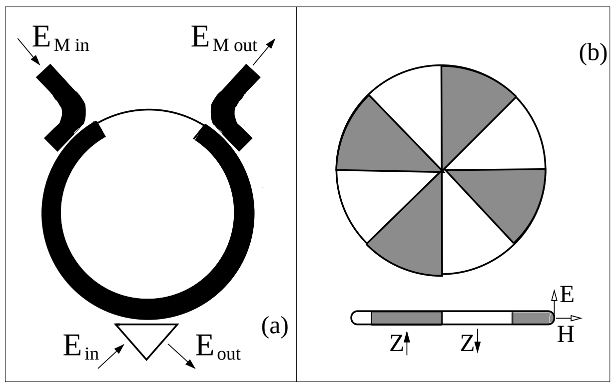

Figure 7. a) A scheme of travelling wave whispering gallery mode electro-optical modulator. Microwave radiation is sent to the disc WGM dielectric cavity using a stripline waveguide. The light is sent into and retrieved out of the cavity via diamond prism. b) A possible spatial structure of the cavity poling.

A possible scheme for a travelling wave WGM EOM is shown in Fig. (7a). Light is sent into a spheroid optical cavity fabricated from a Z-cut periodically poled $LiNbO_3$ substrate via a coupling diamond prism. The side-wall of the disc cavity is polished such that the cavity becomes a part of an oblate spheroid. The optical cavity is placed between two plates of a microwave waveguide that is pumped with an external microwave source. When the microwave frequency corresponds to the free spectral range of the WGM cavity and poling period of the $LiNbO_3$ is properly adjusted to sustain phase matching of the light and microwaves, the parametric interaction between input monochromatic light and microwaves occurs. Optical sidebands are generated and the outgoing light is modulated. A possible poling structure to achieve interaction between the main sequence of WGMs with the microwaves is depicted in Fig. 7b.

4. SINGLE SIDEBAND ELECTRO-OPTICAL MODULATOR USING A PPLN WGM CAVITY

EOM described in the previous section results in generation of symmetric sidebands shifted from the carrier frequency of the input light on the value of the frequency of the microwave pump. This effect results from a symmetry of the system. The main sequence of the WGMs is almost equidistant in a large enough cavity. Generally, each pair of the neighboring optical modes from the main sequence are phase matched with the microwaves if at least one set of pump-signal-idler is phase matched.

Generation of one modulation sideband instead of two sidebands is sometimes useful for photonics applications such as high density wavelength multiplexing and long haul fiber transmission. When a double sideband signal is send through a fiber, dispersion of the fiber causes each spectral component of the signal to experience a

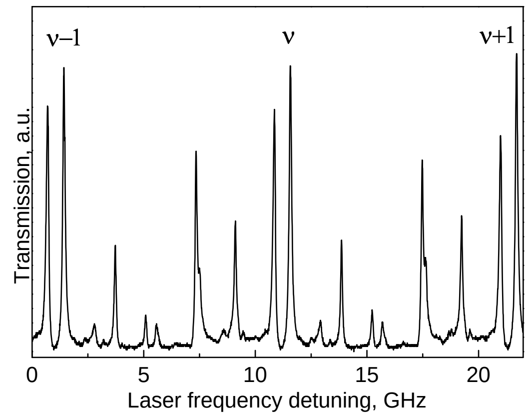

Figure 8. An experimentally measured spectrum of toroidal lithium niobate cavity. The main, nearly equidistant WGM sequence is determined by numbers $(\nu, q = 1, l = 1)$.

different phase shifts. The value of the shifts depend on the transmission distance, modulation frequency, and dispersion properties of the fiber. This effect results in transformation of the amplitude modulated light into phase modulated light and vice versa, that complicates the processing of the received signal significantly.[19, 20] In a single sideband signal transmission this problem is not so stringent.

Single sideband modulators (SSB) and frequency shifters were designed previously for planar systems.[21–27] Here we propose an idea of a novel scheme of SSB EOM using nonequidistant modes of a WGM cavity, which generally possess a large variety of modes. In a spherical cavity there are nonequidistant radial modes. The main sequence of those modes is determined by different numbers of q in Eq. (1). Moreover, in an ideal dielectric sphere a lot of WGMa are degenerate. Changing the shape of the cavity to spheroid results in lifting of the azimuthal degeneracy and appearance of nonequidistant spectrum. A sample of experimental spectrum of 4.6 mm in diameter $LiNbO_3$ oblate spheroid cavity is shown in Fig. 8. Nonequidistant modes can be easily seen in the picture. Once the spheroid is utilized for light modulation, microwave radiation, resonant with two WGMs, might be off-resonant with the other relevant WGMs because of strong nondegeneracy of the spectrum and, therefore, only single sideband generation is possible.

The cavity modes are orthogonal in space, however they overlap geometrically. Wave-functions for the modes with $q = 1, 2, 3$ are shown in Fig. 9. Using periodical poling would allow one to make those modes interacting with microwaves. The interaction energy between the pump, signal and idler (microwaves) modes is similar to Eq. (5) and reads

$$\mathcal{V} = \int_V \chi^{(2)} E_p E_s E_M \; dV, \tag{25}$$

where $\chi^{(2)}$ is varying in space nonlinearity of the cavity, E_p, E_s, and E_M are the pump, signal, and microwave mode amplitudes, V is the volume of the cavity. The periodical poling of lithium niobate with "circular chess structure" (see in Fig. 10b) couples the modes with different values of q. Generally, for any two WGM modes with indexes (ν_1, q_1) and (ν_2, q_2) there exist geometrical two-dimensional poling structure that results in coupling of the modes and the microwaves, resonant with the modes' beatnote.

An important feature of a modulator based on interaction of modes with different q is that the light exits the modes through the coupling prism with slightly different angles. It is possible to spatially separate the generated sideband and the carrier, similarly to acousto-optical modulators. Finally, the efficiency of the interaction is guarantied by almost the same high quality factors of modes with small values of q.

Nondegenerate modes with the same ν and q, but different azimuthal numbers l are orthogonal in space as well. The modes with different l are responsible for less pronounced resonances in Fig. (8). The modes interact with microwaves if the WGM dielectric cavity is fabricated from two crystal wafers with alternating directions of

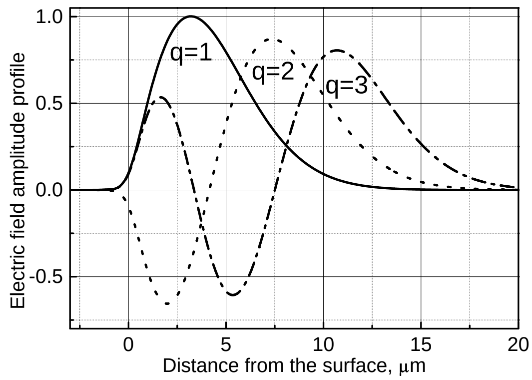

Figure 9. Amplitude profiles of WGMs with different values of q.

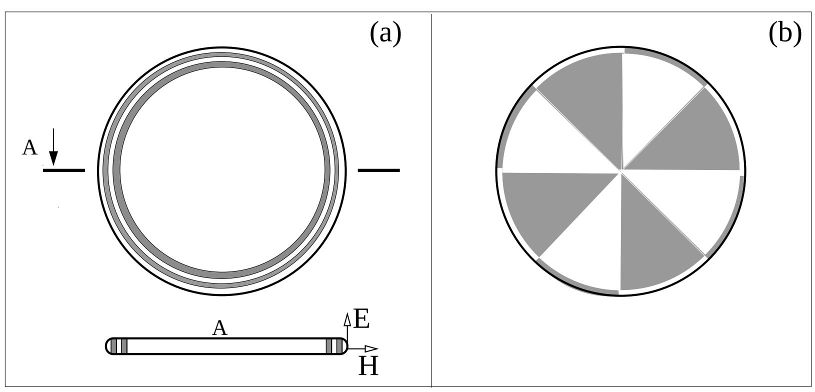

Figure 10. Periodical poling of lithium niobate cavity that allows for interaction of two WGMs with different q and a microwave field. Crossection of the cavity by plane A is shown under the top view of the cavity. Modes of a cavity with poling structure shown in part (a) of the picture need special shape of the microwave waveguide/resonator to have phase matching with the microwaves. Periodically poled cavity shown in (b) may be used in a travelling wave modulator.

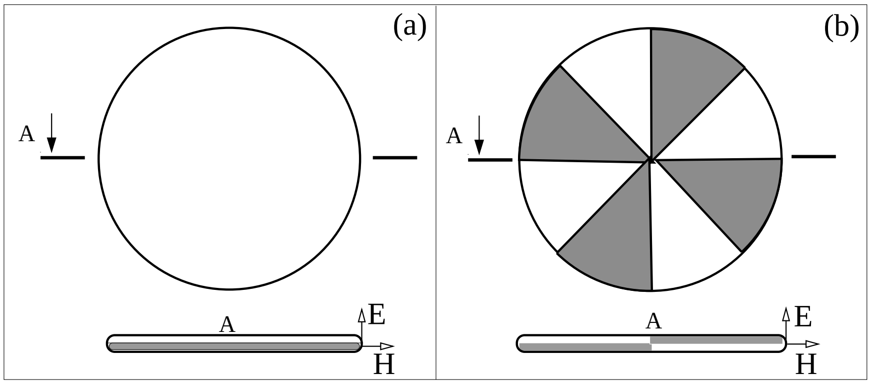

Figure 11. Periodical poling of lithium niobate cavity that results in interaction of two WGMs with different l and a microwave field. Modes of a cavity with poling structure shown in part (a) of the picture need special shape of the microwave waveguide/resonator to have phase matching with the microwaves. Periodically poled cavity shown in (b) may be used in a travelling wave modulator.

the crystal axis (see in Fig. (11)). Again, for any modes with arbitrary l_1 and l_2 it is possible to design a sequence of slices of the crystal to achieve interaction of the light with microwaves. Matching between group velocity of the light and the phase velocity of the microwaves may be created via designing the microwave resonator or waveguide.

5. CONCLUSION

In conclusion, we have proposed to fabricate a whispering gallery mode dielectric cavity from a periodically poled nonlinear material. We show that an appropriate periodical poling leads to the compensation of phase mismatch among cavity modes resulting from cavity material dispersion as well as geometrical cavity dispersion. We show that because of the potentially high quality factors and small volumes of the cavity modes one may create an ultra-low threshold optical parametric oscillator using the cavity. An appropriate periodical poling allows also for an efficient interaction between light and microwaves.

6. ACKNOWLEDGMENTS

The research described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. AAS also acknowledges support from National Research Council.

REFERENCES

1. V. B. Braginsky, M. L. Gorodetsky, and V. S. Ilchenko, "Quality-factor and nonlinear properties of optical whispering gallery modes", Phys. Lett. A **137**, 393-397 (1989).
2. A. J. Campillo, J. D. Eversole, and H. B. Lin, "Cavity quantum electrodynamic enhancement of stimulated emission in microdroplets", Phys. Rev. Lett. **67**, 437-440 (1991).
3. V. S. Ilchenko, X. S. Yao, and L. Maleki, "Microsphere integration in active and passive photonics devices", Proc. SPIE **3930**, 154-162 (2000); Ed. by A. V. Kudryashov and A. H. Paxton.
4. V. S. Ilchenko and L. Maleki, "Novel whispering-gallery resonators for lasers, modulators, and sensors", Proc. SPIE **4270**, 120-130 (2001); Ed. by A. V. Kudryashov and A. H. Paxton.
5. D. A. Cohen and A. F. J. Levi, "Microphotonic millimetre-wave receiver architecture", Electron. Lett. **37**, 37-38 (2001).
6. D. A. Cohen, M. Hossein-Zadeh, and A. F. J. Levi, "Microphotonic modulator for microwave receiver", Electron. Lett. **37**, 300-301 (2001).

7. V. S. Ilchenko, A. A. Savchenkov, A. B. Matsko, and L. Maleki, "Sub-microWatt photonic microwave receiver", IEEE Photon. Techn. Lett. **14**, 1602-1604 (2002).
8. V. S. Ilchenko, A. A. Savchenkov, A. B. Matsko, and L. Maleki, "Whispering gallery mode electro-optic modulator and photonic microwave receiver", J. Opt. Soc. Am. B **20(2)**, (2003); to be published.
9. V. S. Ilchenko, A. B. Matsko, A. A. Savchenkov, and L. Maleki, "High efficiency microwave and millimeter-wave electro-optical modulation with whispering-gallery resonators", Proc. SPIE **4629**, 158-163 (2002), Ed. by A. V. Kudryashov.
10. L. E. Myers, R. C. Eckardt, M. M. Fejer, R. L. Byer, W. R. Bosenberg, and J. W. Pierce, "Quasi-phase-matched optical parametric oscillators in bulk periodically poled $LiNbO_3$", J. Opt. Soc. Am. B **12**, 2102-2116 (1995).
11. M. E. Crenshaw and C. M. Bowden, "Quantum optics of two-level atoms in a dielectric: comparison of macroscopic and microscopic quantizations of the dielectric", Opt. Commun. **203**, 115-124 (2002).
12. M. Martinelli, K. S. Zhang, T. Coudreau, A. Maitre, and C. Fabre, "Ultra-low threshold CW triply resonant OPO in the near infrared using periodically poled lithium niobate", J. Opt. A **3**, 300-303 (2001).
13. D. K. Serkland, R. C. Eckardt, and R. L. Byer, "Continuous-wave total-internal-reflection optical parametric oscillator pumped at 1064 nm", Opt. Lett. **19**, 1046-1048 (1994).
14. R. C. Alferness, S. K. Korotky, and E. A. J. Marcatili, "Velocity matching techniques for integrated optic traveling wave switch/modulators", IEEE J. Quantum Electron. **QE 20**, 301-309 (1984).
15. H. Murata, K. Kinoshita, G. Miyaji, A. Morimoto, and T. Kobayashi, "Quasi-velocity-matched $LiTaO_3$ guided-wave optical phase modulator for integrated ultrashort optical pulse generators", Electron. Lett. **36**, 1459-1460 (2000).
16. H. Murata, A. Morimoto, T. Kobayashi, and S. Yamamoto, "Optical pulse generation by using quasi-velocity-matched guided-wave electrooptic phase modulator", Opt. Quant. Electron. **33**, 785-794 (2001).
17. Y. Q. Lu, M. Xiao, and G. J. Salamo, "Wide-bandwidth high-frequency electro-optic modulator based on periodically poled $LiNbO_3$", Appl. Phys. Lett. **78**, 1035-1037 (2001).
18. K. W. Chang, A. C. Chiang, T. C. Lin, B. C. Wong, Y. H. Chen, and Y. C. Huang, "Simultaneous wavelength conversion and amplitude modulation in a monolithic periodically-poled lithium niobate", Opt. Commun. **203**, 163-168 (2002).
19. G. J. Meslener, "Chromatic dispersion induced distortion of modulated monochromatic light employing direct detection", IEEE J. Quantum Electron. **QE 20**, 1208-1216 (1984).
20. H. Schmuck, "Comparison of optical millimetre-wave system concepts with regard to chromatic dispersion", Electron. Lett. **31**, 1848-1849 (1995).
21. M. Izutsu, S. Shikama, and T. Sueta, "Integrated optical SSB modulator/frequency shifter", IEEE J. Quantum Electron. **QE 17**, 2225-2227 (1981).
22. M. Izutsu, "Light SSB modulators for dense transmission systems", Proc. SPIE **4532**, 28-29 (2001).
23. K. Yonenaga and N. Takachio, "A fiber chromatic dispersion compensation technique with an optical SSB transmission in optical homodyne detection systems", IEEE Photonics Technol. Lett. **5**, 949-951 (1993).
24. R. Stolte and R. Ulrich, "Integrated-optical gigagertz frequency shifter for 1.5 μm signals", Electron. Lett. **33**, 1217-1218 (1997).
25. G. H. Smith, D. Novak, and Z. Ahmed, "Technique for optical SSB generation to overcome dispersion penalties in fibre-radio systems", Electron. Lett. **33**, 1159-1160 (1997).
26. S. Shimotsu, S. Oikawa, T. Saitou, N. Mitsugi, K. Kubodera, T. Kawanishi, and M. Izutsu, "Single side-band modulation performance of a $LiNbO_3$ integrated modulator consisting of four-phase modulator waveguides", IEEE Photonics Technol. Lett. **13**, 364-366 (2001).
27. T. Kawanishi, S. Oikawa, K. Higuma, and M. Izutsu, "Electrically tunable delay line using an optical single-side-band modulator", IEEE Photonics Technol. Lett. **14**, 1454-1456 (2002).